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# Numerical analysis of the flow pathlines in thermo-acoustic couples

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## Abstract

Thermo-acoustic systems use a high amplitude sound-wave for refrigeration or electricity generation without the drawbacks of expensive construction, adverse environmental impact or high maintenance cost. The effective conversion of energy occurs within the “stack” considered as the heart of the system. The time-averaged rate of heat transfer across the edges of the stack is a good indicator of an effective performance. Hence, studying the effect of the geometry of the stack edges together with their locations is useful. Furthermore, current manufacturing practices make it possible to develop diverse stack edges, resulting in an improved efficiency of the heat transfer. For effective modelling of the heat transfer rate, a second-order, double-precision discretization of state variables and a laminar viscous model was used. A numerical model was developed using the commercial code FLUENT. The evolution of the flow vortices at different drive ratio was analyzed. Two edges shapes were considered namely rectangular and rounded edges. Using numerical analysis, this study has pointed out that stack edge profiles has a significant effect on the overall performance of thermo-acoustic systems. Rounding the stack edge profile appears to be beneficial for the system performance. This study point out the link between the non-linearity observed in thermo-acoustic systems, the flow streaming and the mean vorticity at the stack edges.

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**Keywords:** Thermo-acoustic, heat transfer, stack, flow streaming

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## 1. Introduction

The idea of considering thermo-acoustic refrigeration systems is gaining more ground because of the growing concern over negative environmental impact of traditional refrigeration systems. The rapid improvement of the designing of these systems have made their efficiencies comparable to traditional refrigeration systems. In order to refrigerate, thermo-acoustic systems uses appealing input energy sources such as hot exhaust gas streams or solar collectors. Detailed description about the designing and the working of thermo-acoustic systems are available in Ref. [1].

When designing the devices, most of the existing approach relies heavily on the linear acoustic theory, especially the computer program DeltaEC which is used as a predicting tools in many useful studies [2]. However, the accuracy

of this linear approach deteriorates at higher operating states. This leads to the use of higher-order numerical models in order to understand the various loss mechanism during the devices operation. In addition, the interdependence between the pressure, the temperature, the velocity and their derivatives requires an unsteady formulation that takes into account the conservation of momentum, continuity and energy of the solid structures/working fluid [3].

Ishikawa and Mee have simulated the energy transport near thermo-acoustic couples using a Navier-stokes solver [4]. This study demonstrates the existence of heat pumping only near the edges of thermo-acoustic couples (defined as stack much shorter than the acoustic wavelength). This study suggests that a reduction of the plate spacing is going to reduce significantly the vortical motion outside the plates. Bailliet et al. pointed out that “Rayleigh streaming” (or “vortex-like streaming”) plays an important role in the performance of thermo-acoustic devices [5]. To model the unsteady flow at the stack edges, Worlikar and Knio use the Navier-stokes equation formulated as a 2D model [7]. This study demonstrates the existence of vortical structures at the entrance and exit of the stack plates. It appears that the heat transfer from the gas to the plates and the net friction on the plates are closely linked to the evolution of these vortical structures. The use of computational approaches to model the flow field in thermo-acoustic stacks was validated experimentally by Blanc-Benon et al. [8]. Although the study was conducted at low-drive-ratio, close agreement has been reported between the experimental and computational results related to the instantaneous velocity and vorticity fields. Zoonjens et al. numerical study shows that the stack plate half-thickness has a strong influence on the flow structure and heat flux distribution at the plate’s edges [3]. It appears that this plate thickness actually controls the generation of vortices around the stack region. Interestingly, in a different study conducted by the same authors shows that an improvement of the cooling rate and coefficient of performance of thermo-acoustic refrigerator could be achieved by rounding the edge of a rectangular stack section [9].

In a relatively recent study, Abd El-Rahman et al. proposes the use of a 3D computational fluid dynamics models [10]. This study gives some insight into the development of acoustic non-linearities that explain the discrepancies observed using linear acoustic theory. In addition, the flow streaming and the vorticity field have been analyzed. Building upon this study, two 3D computational fluid mechanic model have been analyzed in this paper. In order to obtain the mean velocity components and estimate the flow characteristics at the stack extremities, rectangular and rounded edges were considered. Modern manufacturing techniques allow for the design and the construction for different stack profiles. In addition, the influence of the drive ratio on the induced non-linear effect is analyzed.

## 2. Numerical model

This study refers to the previous conducted analysis referring to the performance of thermo-acoustic couples. The two geometrical configuration of the stack extremities considered have been analyzed in two steps:

- The influence of the drive ratio ( $DR = |p_1|/p_m$ ) on the flow vortices at the stack ends have been analyzed considering rectangular edges profiles;
- Furthermore, the flow velocity distribution around a single rectangular and rounded edge have been compared.

This model consist of a closed-end half-wavelength standing wave system. Helium at 10 kPa filled the resonator tube. A parallel-plate is positioned closer to one end of the resonator tube. The total length of the resonator tube is 5.04 m. The flow conditions and material properties used for all computational runs are provided in Table 1.

A half-wavelength was considered to define the computational domain. A 2D view of the computational domain is shown in Fig. 1. The Boundary conditions are summarized by Equations 1 to 5. The parameters  $u$ ,  $v$  and  $x'$  are respectively the components in the  $x$ ,  $y$  direction and the axial distance from the center of the duct.

- a. Axial “Wall” boundaries

$$u = 0; v=0 \text{ and } dT/dy = 0 \quad (1)$$

- b. Transverse “Wall” boundaries

$$u = 0; v=0 \text{ and } dT/dy = 0 \quad (2)$$

- c. “HX” boundaries

$$u = 0; v=0 \quad (3)$$

- d. “SYM” boundaries

$$v=0 \text{ and } dT/dy = 0 \quad (4)$$

- e. “INLET” boundaries

$$p = p_m + \text{Re}[|p_1|e^{j(\omega t + kx' - \pi/2)}] \quad (5)$$

The copper stack region was treated as two surface groups: a cold region having a negative heat flux of  $-100 \text{ W/m}^2$  and positive region having a positive heat flux of  $100 \text{ W/m}^2$  (Fig. 2).

Table 1. Flow conditions adopted for all simulation runs.

Property	Value	Units
Operating frequency, $f$	100	Hz
Ambient temperature, $T_m$	300	K
Mean pressure	10	kPa
<b>Gas properties</b>		
Prandtl number, $Pr$	0.69	
Thermal conductivity, $k_0$	0.149	W/mK
Heat capacity, $c_p$	5200	J/kgK
Dynamic viscosity, $\mu$	$2.01 \times 10^{-5}$	kg/ms
Kinematic viscosity, $\nu$	$1.24 \times 10^{-4}$	$\text{m}^2/\text{s}$
Ratio of specific heats, $\gamma$	1.665	

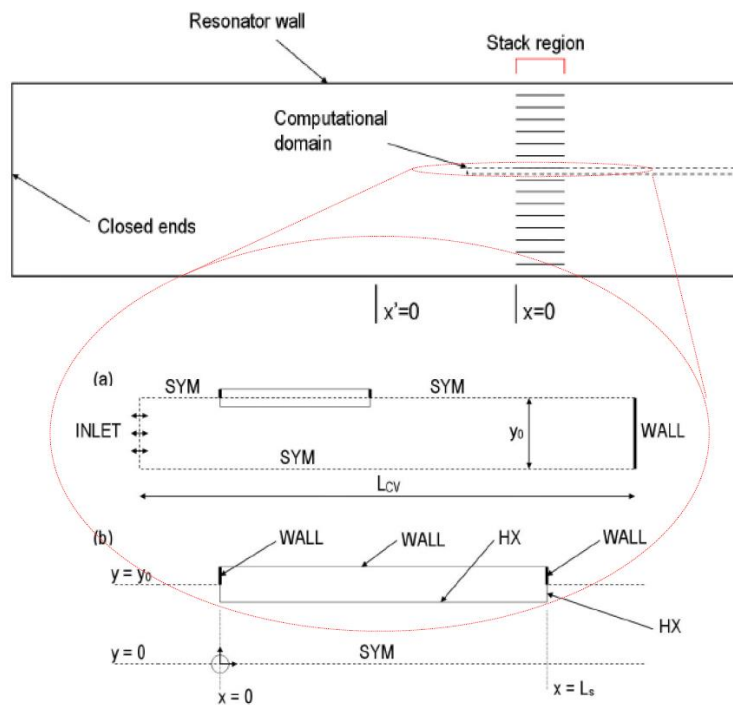


Fig. 1. Computational domain

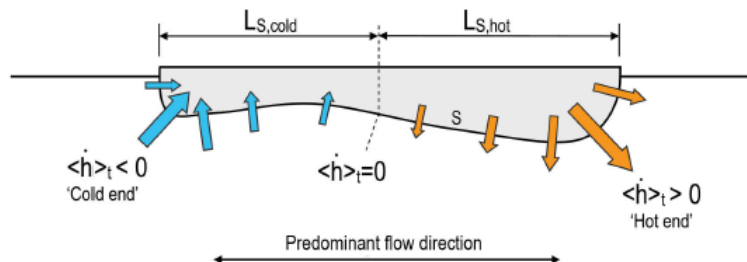


Fig. 2. Thermo-acoustic couple with arbitrary shape for illustration of time-averaged heat transfer surface

### 3. Results and discussions

Examples of 3D computational flow streaming corresponding to one rounded and one rectangular stack edge profile are shown in Fig. 3. All the geometry considered in this analysis have an overall plate length of 0.5 m and a constant thickness of 2.4 mm in order to compare the results obtained.

#### 3.1. Flow streaming at different DR

Fig. 4, Fig. 5 and Fig. 6 show the flow streaming corresponding to a drive ratio (DR) of 1.7%, 3.4% and 5.3% respectively. In order to visualize the flow structure and follow the streamlines generated after each runs, pathlines have been used for visualization. The maximum velocity streamlines for the three DR considered is 1 684 m/s. The flow direction is from the left inlet. In order to evaluate the scales of the vortices generated after each runs, a scale positioned next to each figure has been used. As the fluid moves to the right, growing recirculation zones are formed to the point of dominating the flow structure at the right side of the stack edge (Fig. 4-6). The prevailing flow velocity decreases significantly within the recirculation zones. A similar behavior is observed irrespective of the DR. A slight deceleration has been observed at the downstream of the stack as the flow changes direction. With respect to the DR, the results shown in Fig. 4-6 predicts that vortical pathlines at the larger drive ratio (as considered in this study) are relatively more elongated. It appears that as the DR increases, the vortical pathlines detached itself from the stack extremities accordingly. This can be observed by comparing the three different results reported in the three Figures.

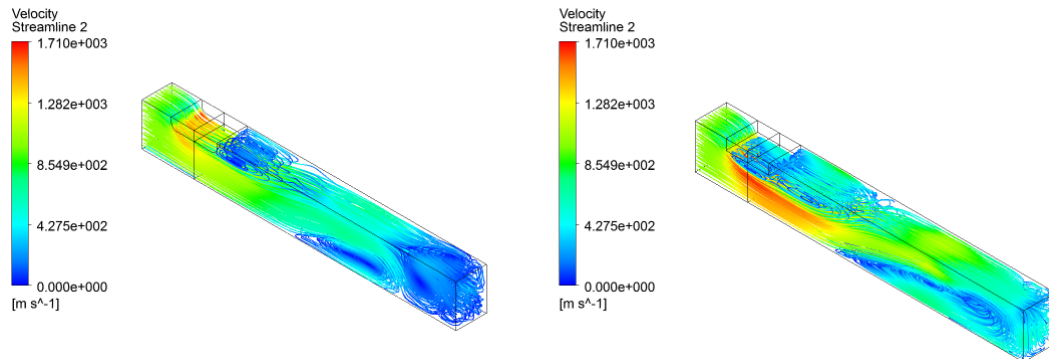


Fig. 3. 3D representation of the flow streaming

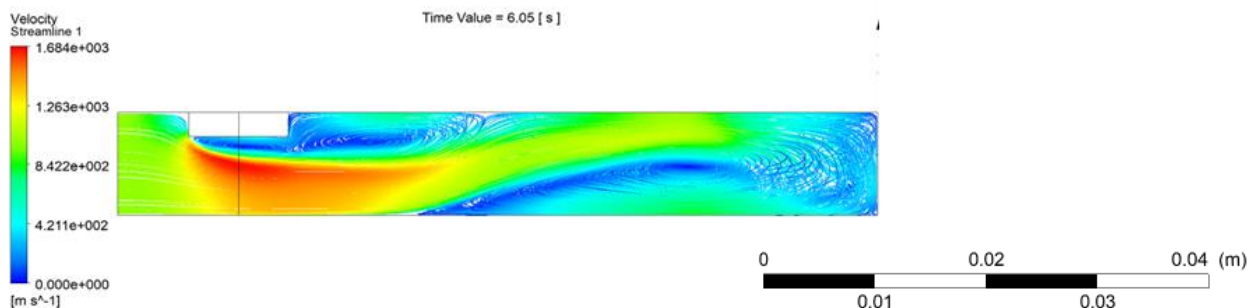


Fig. 4. Flow streaming: DR: 1.7%

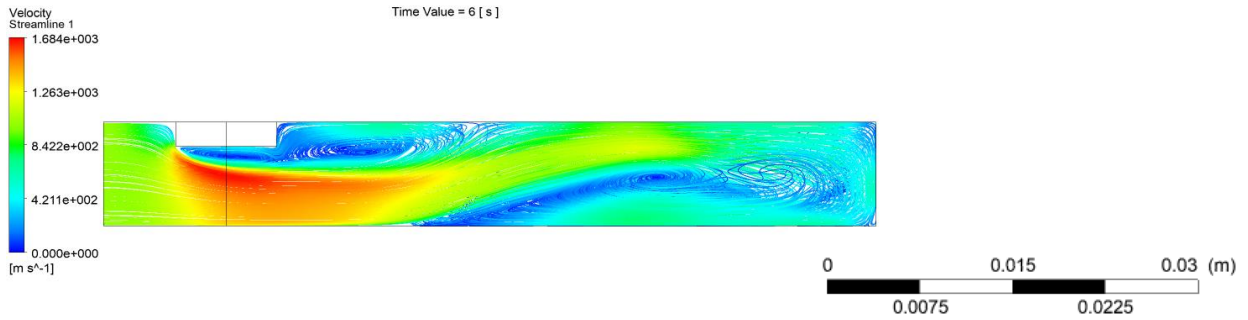


Fig. 5. Flow streaming: DR: 3.4%

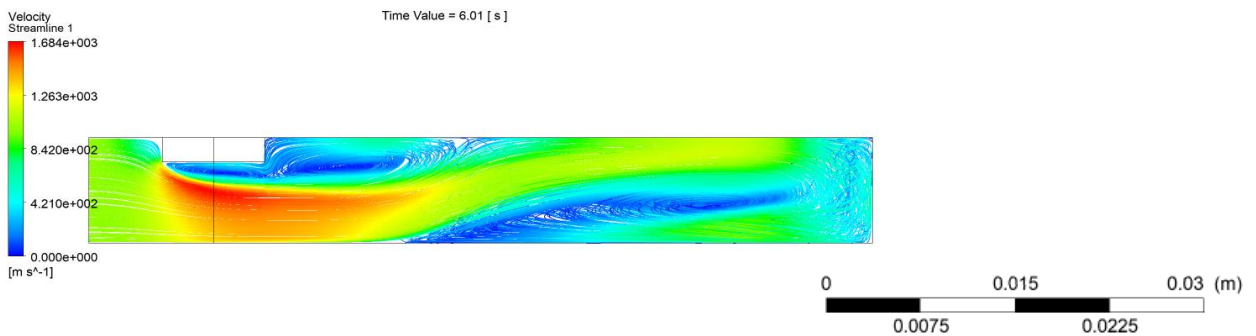


Fig. 6. Flow streaming: DR: 5.3%

### 3.2. Flow streaming for diverse stack edge geometry

In this section, the flow velocity distribution considering rectangular and rounded edges profiles have been analyzed. In order to compare the results obtained, a DR of 6.8 % has been considered for these runs. Fig. 7 shows the flow streaming around the rounded stack edge profile. The maximum velocity was evaluated to be 1 710 m/s. Unlike the results reported in Fig. 8, the recirculation zone size is reduced significantly. Zoontjens et al. [9] pointed out that rounding the stack edge profile decreases the flow resistance which is beneficial for the performance of the entire thermo-acoustic refrigerator. This results suggests that rounded stack edge profiles could potentially improve the performance of thermo-acoustic refrigerators by decreasing the negative effect of acoustic streaming. Hence, abrasive techniques could potentially be used to manufacture this rounded edge profile.

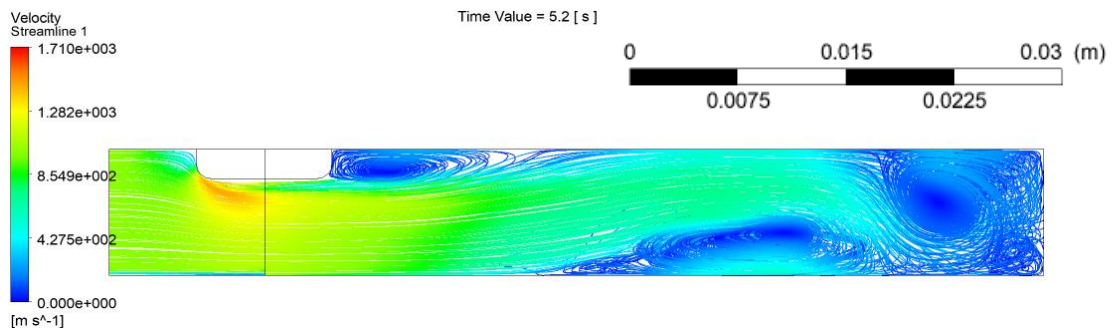


Fig. 7. Flow streaming for rounded stack edge profile

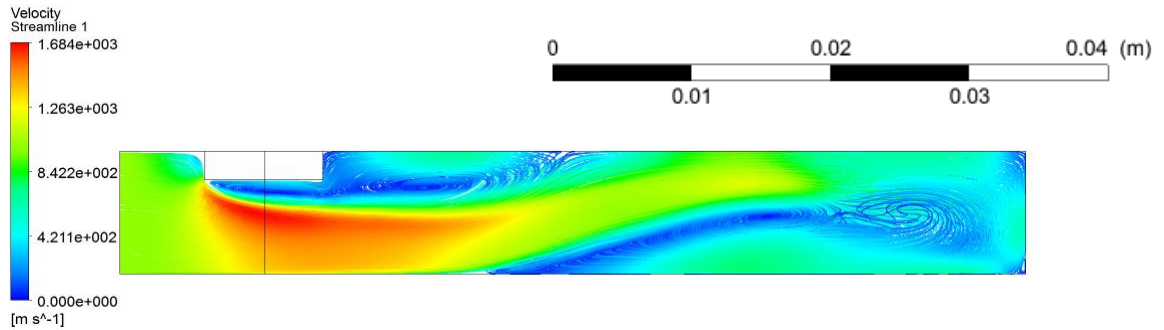


Fig. 8. Flow streaming for rectangular stack edge profile

## Conclusion

In this paper, a numerical analysis of the flow pathlines was performed using the commercial CFD software FLUENT. In order to investigate the influence of the drive ratio on the flow pathlines, three different runs corresponding to 1.7 %, 3.4 % and 5.3 % were performed to predict the flow streaming in each configuration. The results obtained, predicts that vortical pathlines at the larger drive ratio (as considered in this study) are relatively more elongated. It appears that as the DR increases, the vortical pathlines detached itself from the stack extremities accordingly. Which could explain the non-linear effect and the discrepancies observed when analyzing thermo-acoustic refrigerators at higher drive ratio. The comparison between the stack edge geometry shows that rounding the edge profile reduces the recirculation zone size. Hence, this explains why the flow resistance due to the acoustic streaming is better with such edge geometry. It appears that the use of abrasive techniques for the manufacturing of rounded stack edge profiles could enhance the heat transfer and ultimately improve the performance of thermo-acoustic systems.

## Acknowledgements

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